THE TRANSFORMATION OF THE WAVE HEIGHT DURING SHORE-BREAKING: THE ALPHA WAVE PEAKING PROCESS

by

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FOREWORD

This work presents a numerical solution for the predicton of wave height behavior during the littoral wave process of shore-breaking. It is basic support methodology required in the develpment of a multiple shore-breaking wave transformation model described in subsequent work.

The work described herein constitutes partial fulfillment of contractual obligations with the Federal Coastal Zone Management Program (Coastal Zone Management Act of 1972, as amenoed) through the Florida Office of Coastal Management subject to provisions of contract CM-37 entitled "Engineering Support Enhancement Program". Under provisions of DNR contract C0037, this work was reviewed by the Beaches and Shores Resource Center, Institute of Science and Public Affairs, Florida State University. The document has been adopted as a Beaches and Shores Technical and Design Memorandum in accordance with provisions of Chapter 16B-33, Florida Administrative Code.

At the time of submission for contractural compliance, James H. Balsillie was the contract manager and Administrator of the Analysis/Research Section, Hal N. Bean was Chief of the Bureau of Coastal Data Acquisition, Deborah E. Flack Director of the Division of Beaches and Shores, and Dr. Elton J. Gissendanner the Executive Director of the Florida Department of Natural Resources.

Deborah E. Flack

Deborah E. Flack, Director
Division of Beaches and Shores

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ABSTRACT

As waves begin to shore-break, the wave crest often tends to rapidly increase in height, reaching a maximum at the shore-breaking position. This phenomenon, termed alpha wave peaking, is primarily dependent on the wave steepness and may be predicted according to:

$$\frac{H_b}{H_i} = 1.0 - 0.4 \ln \left[\tanh \left(100 \frac{H_i}{g\, T^2} \right) \right]$$

where H_i is the incident wave height, T is the wave period and H_b is the shore-breaking wave height. Transformation of H/H_i, where H is the local wave height, is given by:

$$\frac{H}{H_i} = \frac{H_b}{H_i} - \Phi_2 \left\{ \tanh \left[\Phi_1 \left(\frac{d}{H} - \frac{d_b}{H_b} \right) \right] \right\}^{0.7}$$

in which d_b is the water depth at shore-breaking, d is the local water depth, and solutions for Φ_1 and Φ_2 are developed in the text.

INTRODUCTION

Generally, as waves near the shoreline, the height of the waves first tend to decrease and then to increase rapidly just prior to shore-breaking. This increase in the wave height beginning just prior to and reaching a maximum at shore-breaking can, even over gentle bed slopes, be "... remarkably sudden ..." (Munk, 1949) and accompanied by progressive distortion and asymmetry of the wave in profile view, has been termed alpha wave peaking by Balsillie (1980). Alpha wave peaking has been observed as characteristic activity in shore-breaking wave mechanics from many studies (Scripps Institute of Oceanography, 1944a, 1944b; Putnam, 1945; Munk, 1949; Iverson, 1952; Stoker, 1957; Kinsman, 1965; Byrne, 1969; Clifton, Hunter and Phillips, 1971; Komar, 1976; Nakamura, Shiriashi and Sasaki, 1966; Van Dorn, 1966; Buhr Hansen and Svendsen, 1979; Balsillie, 1980; etc.).

Two mechanisms occur during shore-breaking: 1. the transformation of H/H_i and 2. the transformation of H'/H, where H is the local wave height, H' is the amount of the wave lying above the design water level (DWL), and H_i is the wave height at the initiation of alpha wave peaking. Pertinent wave height parameters are illustrated in Figure 1. The first of the above mechanisms defines the subject of this paper, the second is the subject of a companion paper (Balsillie, in manuscript).

ANALYTICAL APPROACH

Of presently available theories, Cnoidal wave theory seems to have gained popularity for predicting the transformation of shoaling waves in shallow water. Svendsen and Buhr Hansen (1976) discuss the applicability of Cnoidal theory using developments of Skovgaard et al. (1974), for the deformation of waves up to shore-breaking (the theory is applicable where $d/L_0 < 0.10$ or $d/L < 0.13$, seaward of which they recommend the use of Airy wave theory). Svendsen and Buhr Hansen state "... even though cnoidal theory seems to predict the wave height variation reasonably well, no information can be deduced from that theory (or any other theory) about where breaking occurs." Cnoidal wave theory is not, however, simple nor expedient to apply. First, it requires tabulated coefficients for realization of solutions. Second, and more critically, it requires that the

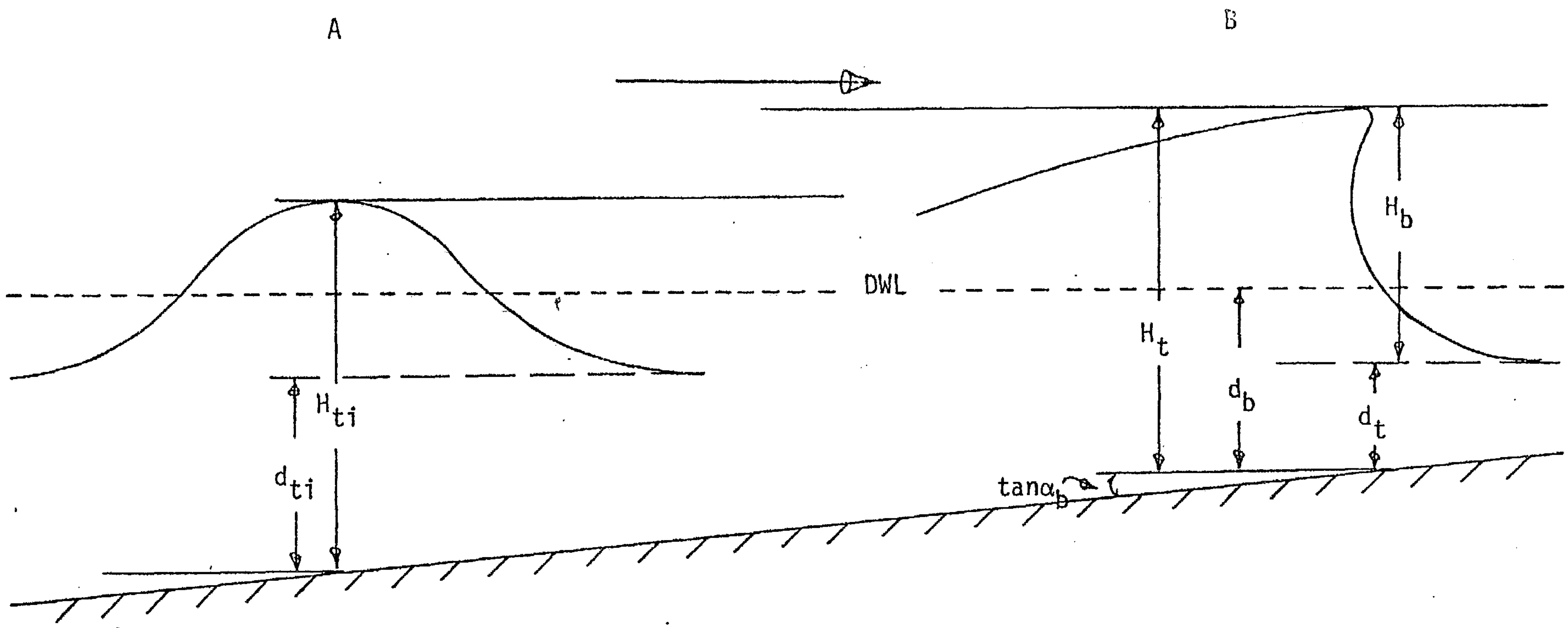


Figure 1. Pertinent nearshore wave height parameters; parameters at A illustrate conditions at the initiation of alpha wave peaking, those at B represent conditions at the shore-breaking position.

local wave length is known when, in fact, the wave length is seldom known short of additional theoretical calculations for an approximation. It is the intent of this work to provide a practicable solution to the problem.

Mathematical descriptions developed in following sections assume that the initial wave height, H_i, and wave period, T, are known. In an earlier study, Balsillie (1980) used H_o or H_m as indicators of H_i, where H_o is the specified deep water wave height, and H_m is the wave height measured in the constant depth portion of a laboratory wave channel. In many laboratory investigations it has been found that initial wave characteristics are in the range where H_o and H_m and the resulting value of H_i are approximately equivalent. Generally, this occurs for waves with higher wave steepness values. However, due to refraction and frictional effects, etc., where the generated wave steepness is small, H_i can become significantly less than H_o or H_m. The importance of the latter phenonmenon is illustrated by an example from the laboratory data of Putnam (1945) in Figure 2. Therefore, in this study only data which describe the continuous transformation of waves across a known bathymetry, from which H_i can be determined, are considered.

In the earlier work of Balsillie (1980), it was reported that the alpha wave peaking parameter, H_b/H_i, is dependent on the equivalent wave steepness parameter, $H_i/(g\ T^2)$, and the bed slope, tan α_b. The influence of these parameters are included in ensuing analyses. In addition, the continuous transformation of H/H_i during shore-breaking is investigated. First, however, determination of where alpha wave peaking is initiated and terminated require identification. Where possible, both field data and laboratory data are considered. It is to be noted, however, that laboratory information by far constitutes the bulk of available data. However, since the study of Balsillie, new laboratory data have become available (see the table). In addition to the study of Putnam (1945), results are reported by

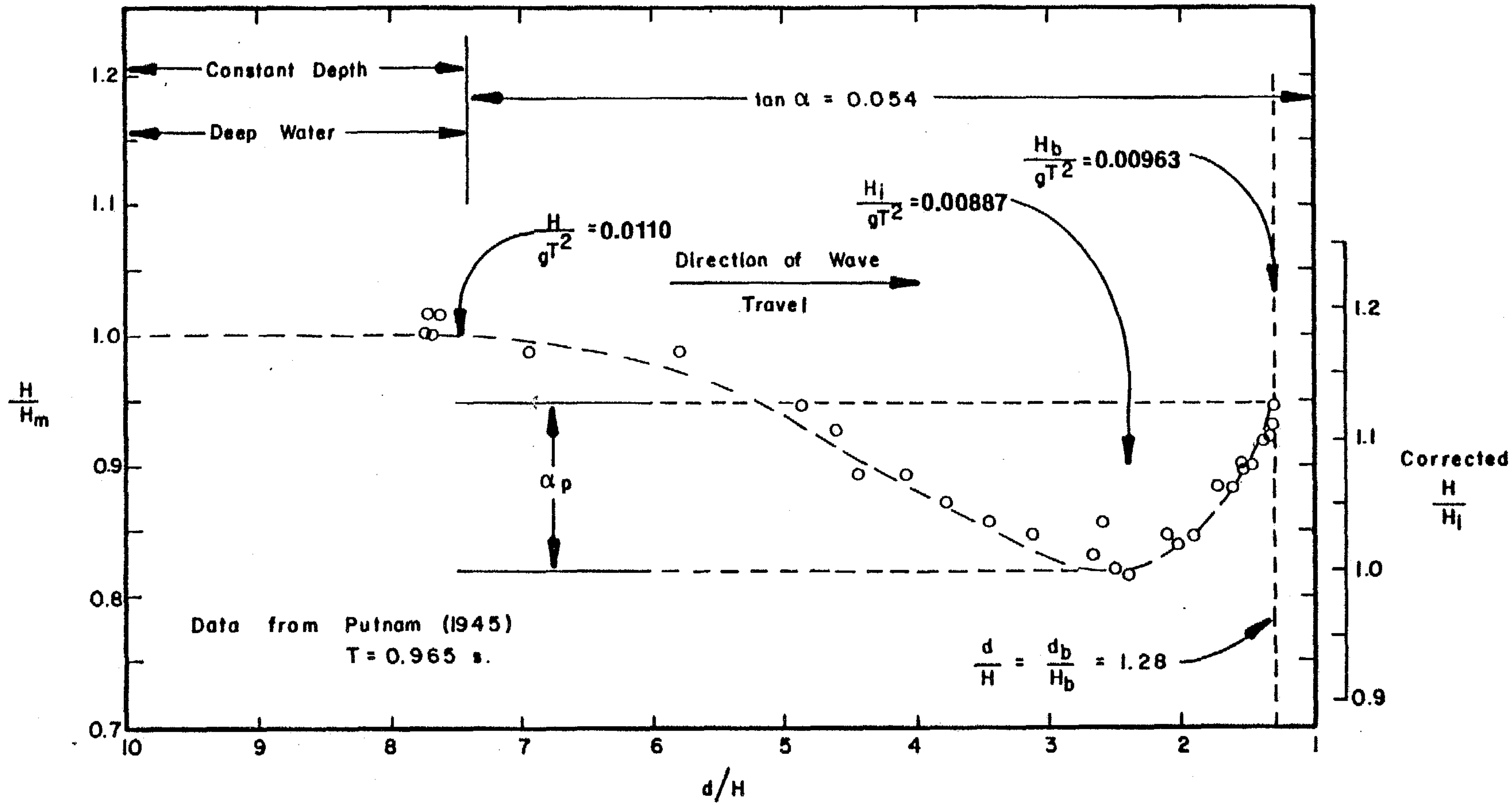


Figure 2. Illustration of wave transformation from deep water to shore-breaking, where the alpha wave peaking process is given the notation α_p.

Table of data used in analyses.

	tan α_b	T (s)	$\frac{H_b}{H_i}$	$\frac{d_i}{H_i}$	$\frac{d_i}{d_b}$	$\left(\frac{H_i}{g\,T^2}\right)^{-1}$	$\left(\frac{H_b}{g\,T^2}\right)^{-1}$
FIELD DATA							
Wood (1970, 1971)*	0.0556	3.49	1.58	4.23	2.00	451.2	286.2
LABORATORY DATA							
Putnam (1945)	0.072	0.865	1.04	2.34	1.76	78.8	74.4
	"	1.15	1.29	3.13	2.27	162.0	131.0
	"	1.22	1.29	2.69	2.67	189.4	147.0
	"	1.50	1.66	4.77	3.17	367.5	236.8
	"	1.54	1.58	4.45	2.92	381.1	267.0
	"	1.97	1.86	5.01	3.11	745.7	462.8
	0.054	0.86	1.08	2.22	1.66	85.3	78.6
	"	0.965	1.11	2.50	1.93	112.7	103.8
	"	1.34	1.48	3.96	2.96	279.3	213.7
	"	1.50	1.58	4.39	3.16	386.8	278.7
	"	1.97	1.84	5.94	3.47	927.6	557.9
	step**	1.05	1.16	----	----	126.2	108.5
	"	1.09	1.19	----	----	142.6	119.4
	"	1.35	1.31	----	----	237.4	181.5
	"	1.50	1.60	----	----	375.0	235.0
	"	1.98	1.86	----	----	997.8	441.0
Buhr Hansen and Svendsen (1979)	0.0292	0.833	1.32	3.64	2.55	207.0	156.3
	"	1.00	1.13	2.57	1.66	104.2	91.8
	"	1.00	1.21	2.95	1.89	153.1	126.4
	"	1.00	1.35	3.92	2.66	258.4	191.8
	"	1.25	1.30	2.50	1.51	162.9	124.3
	"	1.25	1.37	3.15	2.04	229.2	167.6
	"	1.25	1.46	3.95	2.42	395.3	270.1
	"	1.67	1.46	3.57	2.29	283.2	194.8
	"	1.67	1.42	3.57	2.15	302.5	209.8
	"	1.67	1.47	3.84	2.27	340.3	233.5
	"	1.67	1.48	4.13	2.41	389.1	261.3
	"	1.67	1.66	5.05	3.09	675.7	408.2
	"	2.00	1.69	4.47	2.66	608.6	359.3
	"	2.00	1.95	5.50	3.01	1048.2	537.8
	"	2.50	1.84	5.22	2.63	874.9	479.7
	"	2.50	2.20	5.95	2.75	1531.4	704.1
	"	3.33	2.39	6.67	3.08	2544.5	1069.2
Singamsetti and Wind (1980)	0.025	1.28	1.27	----	----	170.8	134.9
	"	1.55	1.25	----	----	173.8	138.8
	0.050	1.038	1.21	----	----	162.4	134.5
	"	1.55	1.20	----	----	162.9	135.5
	"	1.55	1.28	2.43	1.56	216.0	168.2
	"	1.55	1.27	2.68	1.22	210.2	165.8
	0.100	1.035	1.11	----	----	160.3	144.8
	"	1.555	1.25	----	----	173.8	139.3
	0.200	1.038	1.35	----	----	157.6	116.7

*Based on 400 consecutive wave measurements; ** step had a slope of 0.444, post-step slope was 0.009.

Buhr Hansen and Svendsen (1979) and Singamsetti and Wind (1980) to represent a wide range in bed slope conditions.

TERMINAL BOUNDARY CONDITIONS

The terminal boundary of alpha wave peaking is defined as the shore-breaking point. Galvin (1968) provides a comprehensive description of the various types of shore-breaking waves. Of the principal types, however, spilling and plunging shore-breakers constitute those more commonly applied in design considerations. The shore-breaking point of a plunging breaker is defined to occur when the front face of the wave crest becomes vertical (Figure 1); the shore-breaking point of a spilling breaker occurs when the top of the wave crest becomes unstable and water and foam slides or spills down the front face of the crest.

Two parameters identifying termination of alpha wave peaking are d_b/H_b and H_b/H_i. The first parameter may be straightforwardly given by the McCowan criterion (McCowan, 1894; Munk, 1949; Balsillie, in manuscript), illustrated in Figure 3, and given by:

$$\frac{d_b}{H_b} = 1.28 \tag{1}$$

where H_b is the shore-breaking wave height, and d_b is the water depth at shore-breaking. Enhancement in precision of d_b/H_b prediction has been attempted by incorporating the bed slope and wave steepness. Balsillie (in manuscript) found, however, that equation (1) as yet constitutes the most reliable equation with accuracy limits to the 95% C.I. of ± 0.029 H_b. Additional analysis indicates (Balsillie, in manuscript) that equation (1) applies equally well to both spilling and plunging shore-breakers.

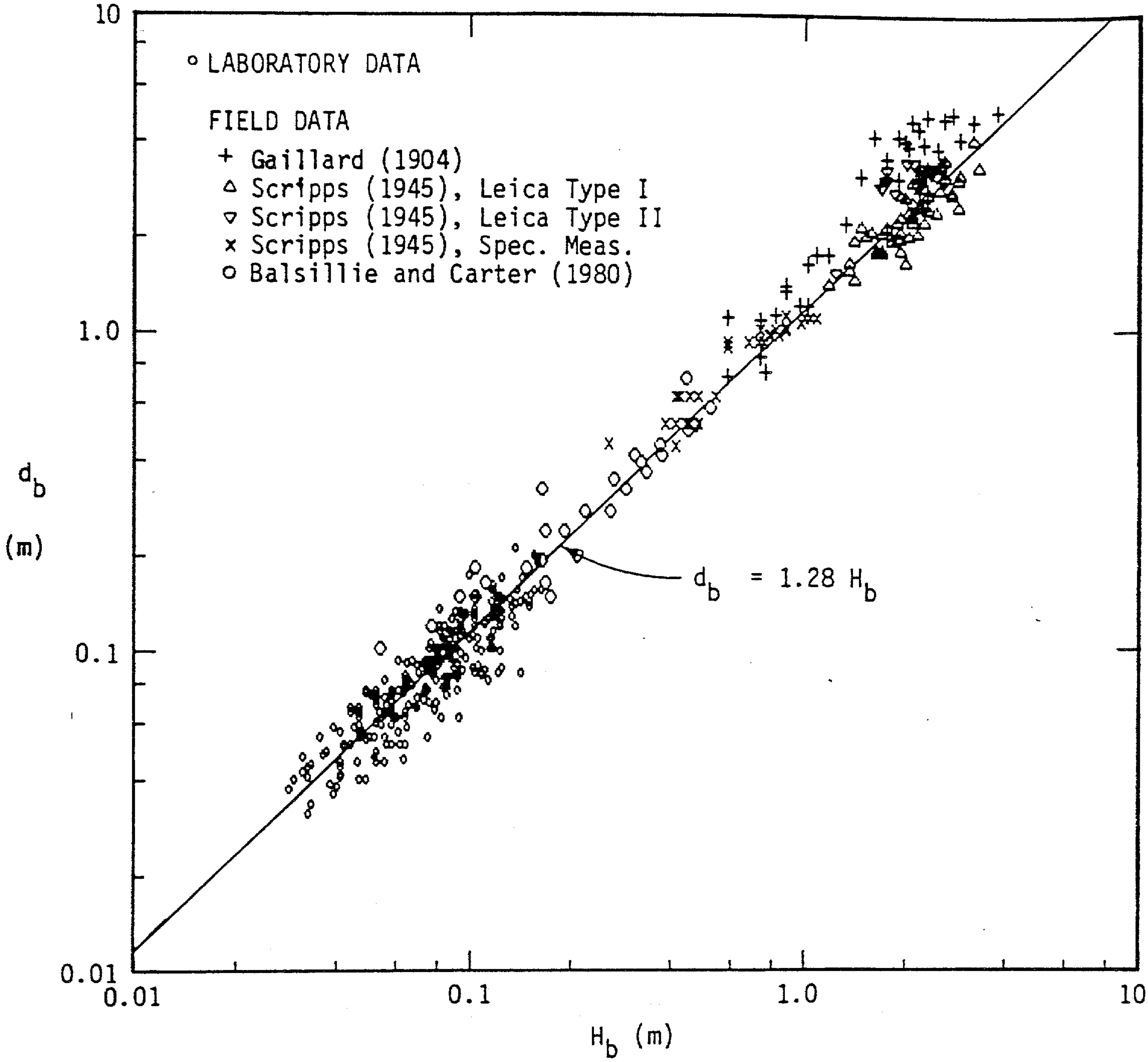


Figure 3. Relationship between the water depth at shore-breaking, d_b, and the shore-breaking wave height, H_b (from Balsillie, in manuscript).

The second parameter, H_b/H_i, describing the relative height attained as a result of alpha wave peaking, is more difficult to quantify. It is, however, considered to be a terminal boundary parameter since H_i is understood to be specified as input.

In the previous work published by the author (Balsillie, in manuscript), both wave steepness and bed slope were indicated to affect alpha wave peaking. However, based on new data, and subsequent and considerable dimensional analyses and testing, the following relationship can be recommended:

$$\frac{H_b}{H_i} = 1.0 - 0.4 \ln \left[\tanh \left(100 \frac{H_i}{g\ T^2} \right) \right] \tag{2}$$

illustrated in Figure 4.

Additional attention was given to the bed slope; no refinement was found to improve equation (2). In fact, equation (2) is evaluated for a wide range of bed slope conditions; scatter might easily be attributed to the difficulty in identifying where shore-breaking occurs. Subsequent work by the author suggests that the bed slope is probably more instrumental in influencing the type of shore-breaker that will be produced.

Due to scale differences between axes of Figure 4, wave steepness data from the table are plotted in Figure 5, where now the axes are comparable. Dividing both sides of equation (2) by $g\ T^2$ yields:

$$\frac{H_b}{g\ T^2} = \frac{H_i}{g\ T^2} \left\{ 1.0 - 0.4 \ln \left[\tanh \left(100 \frac{H_i}{g\ T^2} \right) \right] \right\} \tag{3}$$

which is superimposed upon the data to show good agreement.

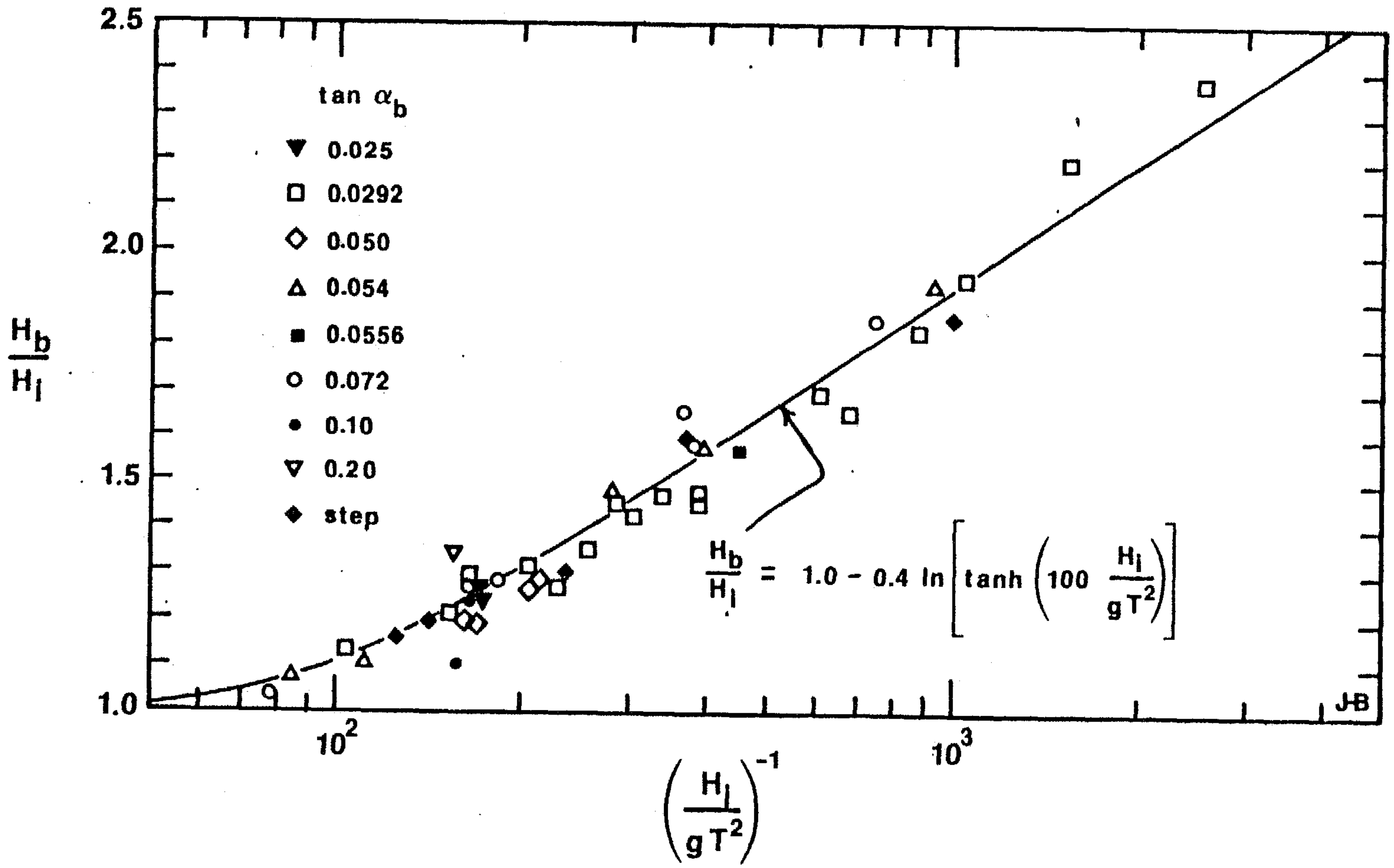


Figure 4. Relationship for prediction of the shore-breaking wave height from the initial equivalent wave steepness parameter.

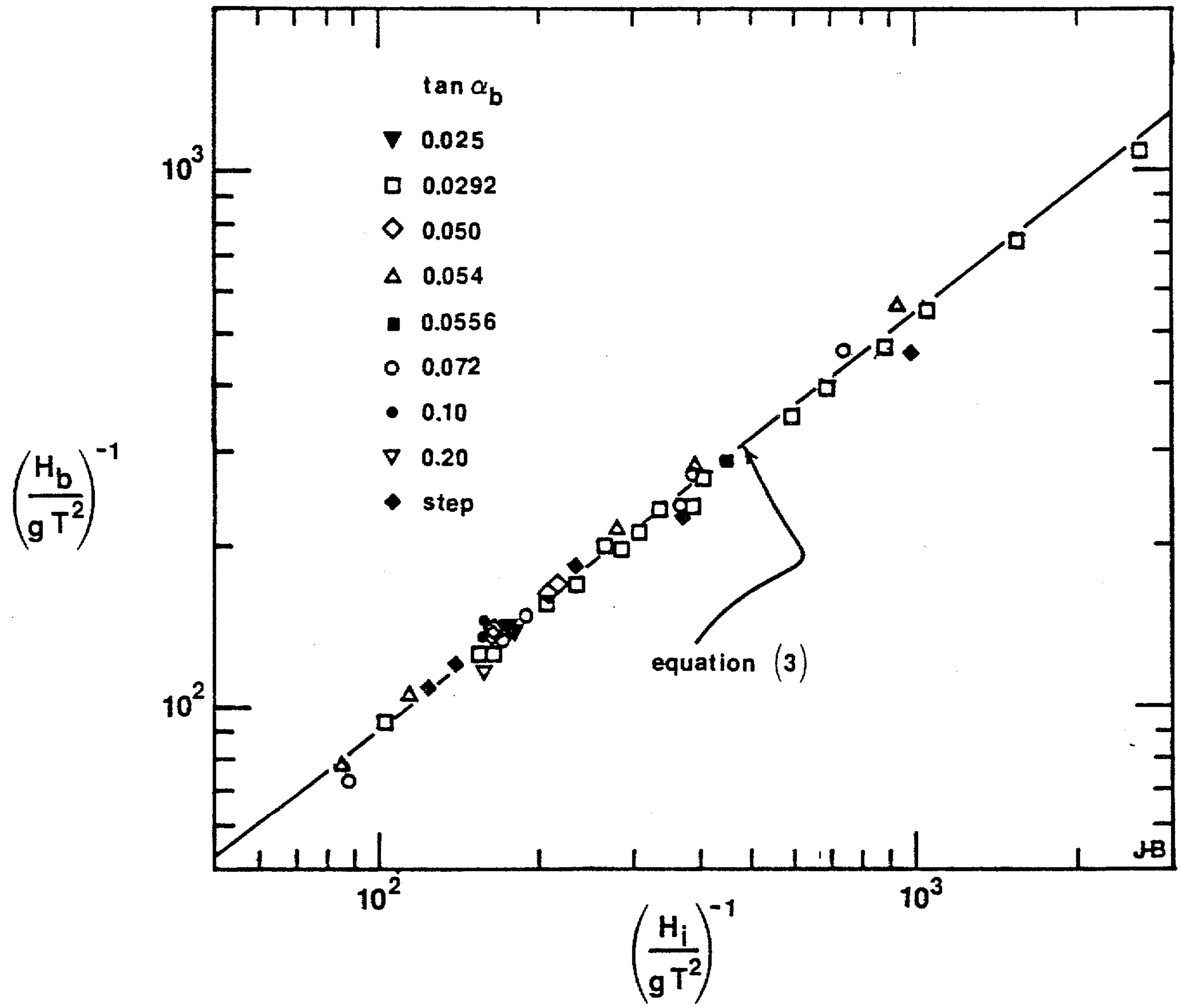


Figure 5. Relationship between the equivalent wave steepness parameter evaluated at initiation of alpha wave peaking and at shore-breaking.

INITIAL BOUNDARY CONDITIONS

Various investigators (e.g., Stokes, 1880; Galvin, 1969; Dean, 1974) have conducted studies to delineate constraints of breaking. It was Munk (1949), however, who considered in some detail wave peaking in the shore-breaking process. He applied the Rayleigh assumption (Eagleson and Dean, 1966) given by:

$$c_i \, E_i \;=\; c_b \, E_b \tag{4}$$

where c is the phase speed (shallow water condition only, where wave period is conserved and no energy is lost) and E is the total wave energy, and the subscripts i and b refer to conditions at initiation of alpha wave peaking and at shore-breaking, respectively. Using Solitary wave theory, Munk (1949) suggests that:

$$\frac{d_i}{d_b} = \left(\frac{H_b}{H_i}\right)^{3/4} \tag{5}$$

Equation (5) is plotted in Figure 6, from which the non-representative nature of the equation is apparent. However, based on what is known about shore-breaking wave activity, it is possible to develop a representative mathematical relationship. It has been demonstrated (Balsillie, in manuscript) that the water depth is the most influencial factor causing shore-breaking. Therefore, the solution may be dependent on d_b/H_b for which there is a solution (Figure 3), and from which it follows that we wish to solve for d_i/H_i rather than d_i/d_b of equation (5), where d_i/H_i must remain larger than d_b/H_b. Of the factors remaining, there may be dependency on the incident wave steepness and/or bed slope. Since required input parameters include the incident wave height and period, the incident wave steepness parameter is considered first. Data from the table are plotted in Figure 7, and the following equation is suggested:

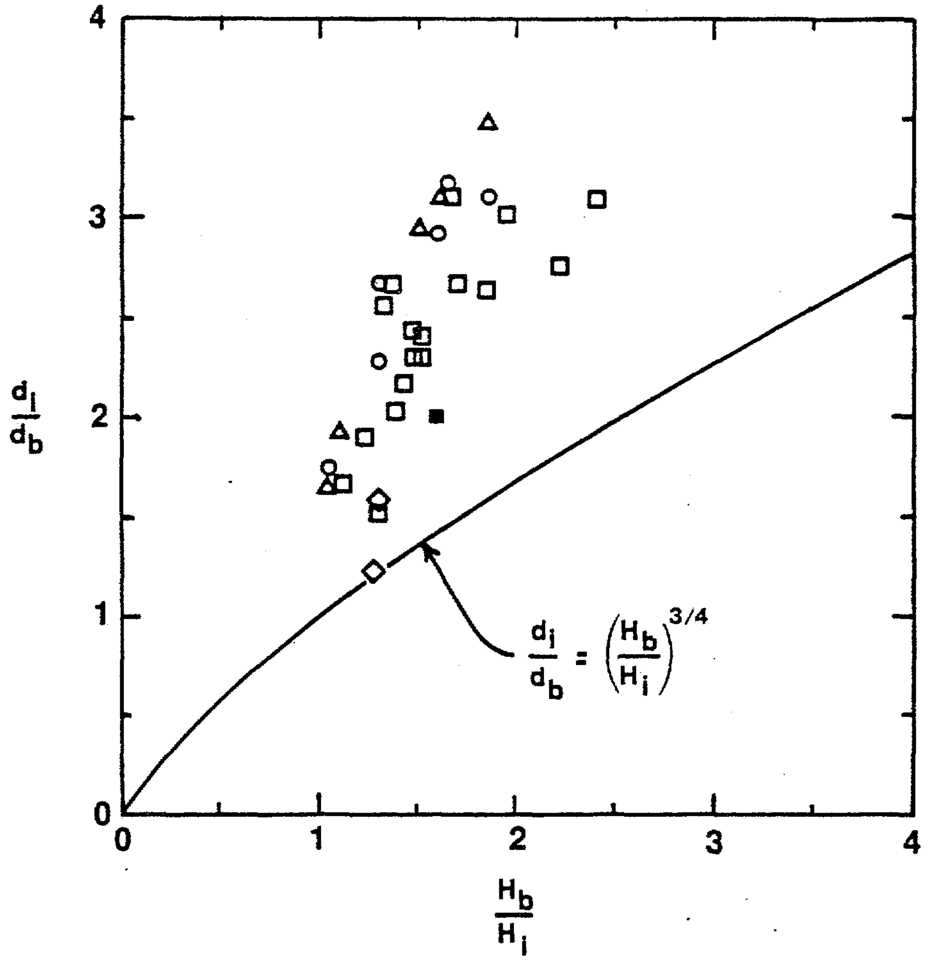


Figure 6. Evaluation of Munk's (1949) parameter for determining the initiation of wave peaking in the shore-breaking process (symbols as for Figure 5).

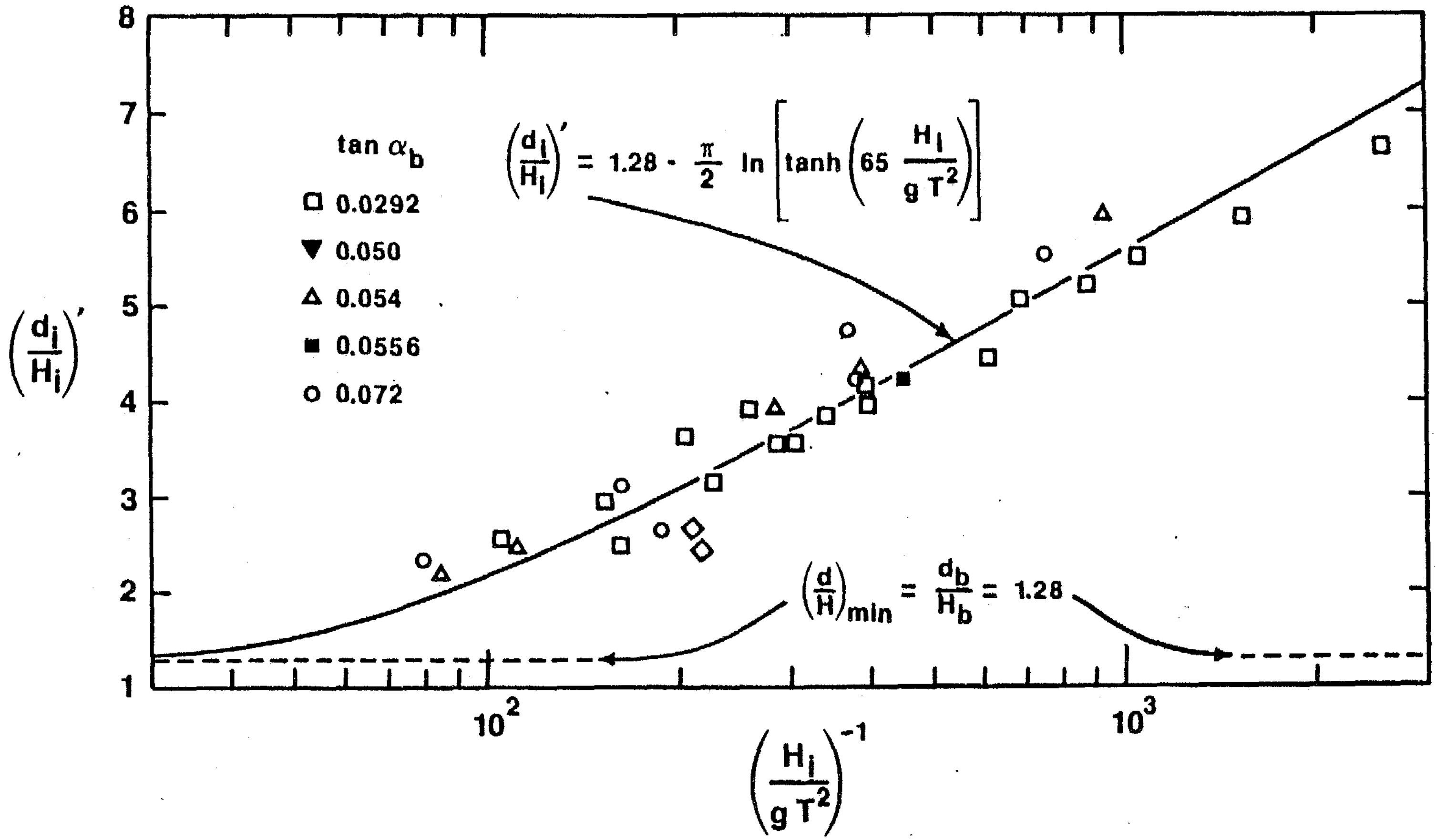


Figure 7. Relationship for the prediction of the relative depth at which alpha wave peaking is initiated.

$$\left(\frac{d_i}{H_i}\right)' = \frac{d_b}{H_b} - \frac{\pi}{2} \ln \left[\tanh \left(65 \frac{H_i}{g\,T^2} \right) \right] \tag{6}$$

in which it is assumed that $d_b/H_b = 1.28$. The equation represents a significant range of bed slopes (i.e., 0.0292 to 0.072) for data from a variety of sources.

TRANSFORMATION OF H/H_i

In addition to specification of the boundary conditions, it is desirable to be able to predict the continuous behavior of alpha wave peaking. Such behavior, for example, may be important in determining horizontal and vertical impact loading potential of shore-breaking waves, and in sediment transport prediction.

Data tabulated by Putnam (1945) and Buhr Hansen and Svendsen (1979) are used to determine the nature of the transformation. The general equation is given by:

$$\frac{H}{H_i} = \frac{H_b}{H_i} - \Phi_2 \left\{ \tanh \left[\Phi_1 \left(\frac{d}{H} - \frac{d_b}{H_b} \right) \right] \right\}^{0.7} \tag{7}$$

where Φ_1 is a coefficient which determines where the transformation of H/H_i begins, given by:

$$\Phi_1 = \frac{e}{(d_i/H_i)' - (d_b/H_b)} \simeq \frac{2.7183}{(d_i/H_i)' - (d_b/H_b)} \tag{8}$$

in which $(d_i/H_i)'$ is given by equation (6), e is the Naperian constant, and Φ_2 determines the local peaked height during shore-breaking given by:

$$\Phi_2 = \frac{H_b}{H_i} - 1.0 \tag{9}$$

in which H_b/H_i is given by equation (2). Equation (7) is evaluated (dashed curves) in Figure 8 for various bed conditions. Data from Singamsetti and Wind (1980) are not plotted because the authors did not tabulate the transformation information. Only four data points are available for the field data of Wood (1970, 1971) and are not plotted. Data from Putnam (1945) for the step slope could be plotted, but would require considerable license in estimation to determine the value of d_i (since the waves began to shore-break on the step slope over which measurements were widely spaced).

In many of the plots of Figure 8, the laboratory data suggest that d_b/H_b is closer to unity than to a value of 1.28. From Figure 3, however, it is evident that laboratory data "tend" toward a lower value. This may be symptomatic of difficulties in determining precisely when small laboratory waves shore-break (i.e., since this must be visually observed and cannot be measured). The terminal boundary condition of $d_b/H_b = 1.28$ is, therefore, maintained. Scatter of data relative to equation (7) is noted in some of the plots. Overall, however, the shape of the transformation appears to be well represented by equation (7).

CLOSURE

The inapplicability of Airy wave theory to represent shallow water waves is well known. Cnoidal wave theory has been recommended (Svendsen and Brink-Kjaer, 1972; Skovgaard _et al_. 1974; Svendsen and Buhr Hansen, 1976) where $d/L_o < 0.10$ or $d/L < 0.13$. However, Cnoidal theory is not easy to apply. First, it requires the use of tabulated ellipitical function. Second, and more problematic, it requires that the local wave length is known when,

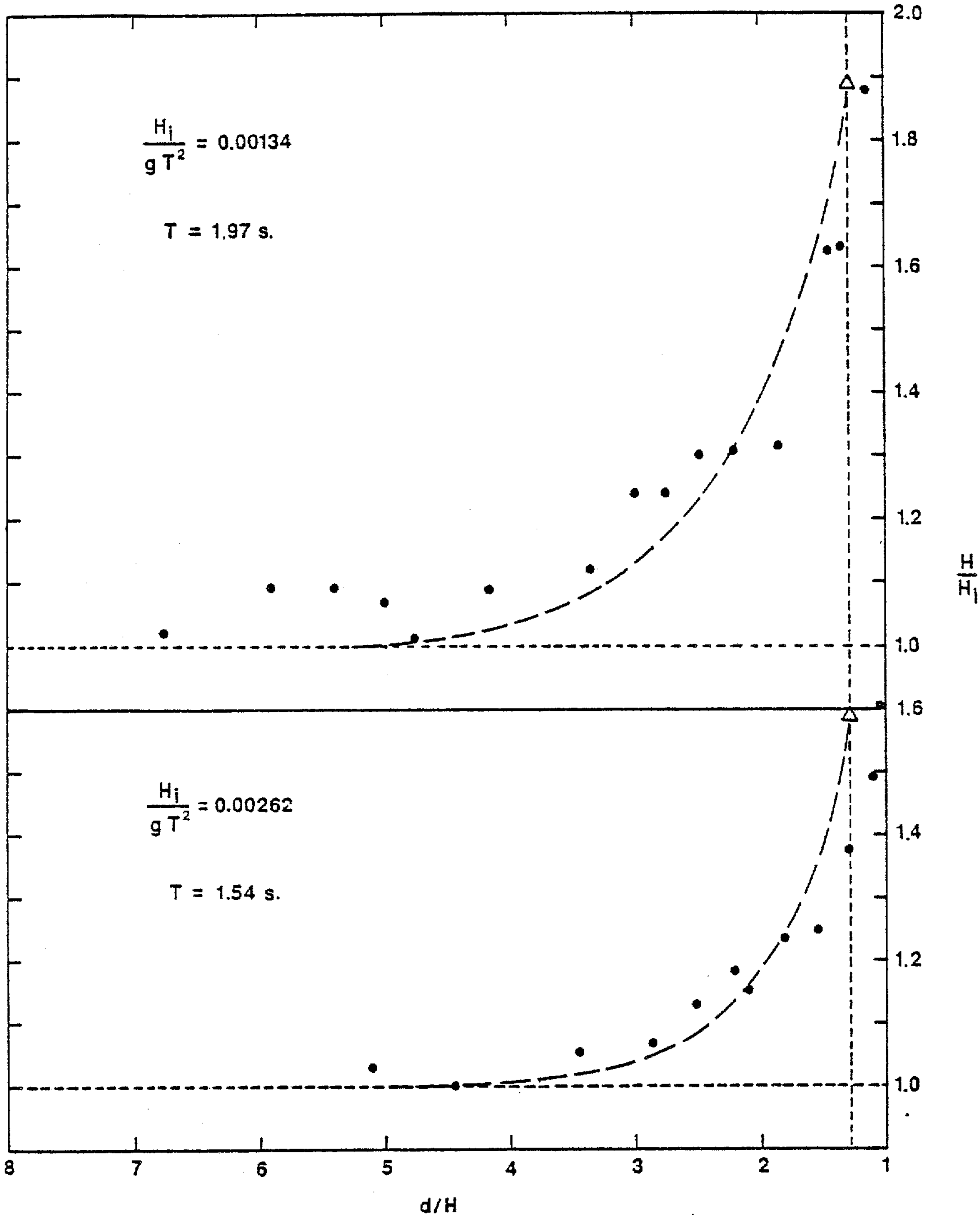


Figure 8a. Alpha wave peaking; data from Putnam (1945) for $\tan \alpha_b$ = 0.072; shore-breaking occurs at Δ where d/H = 1.28.

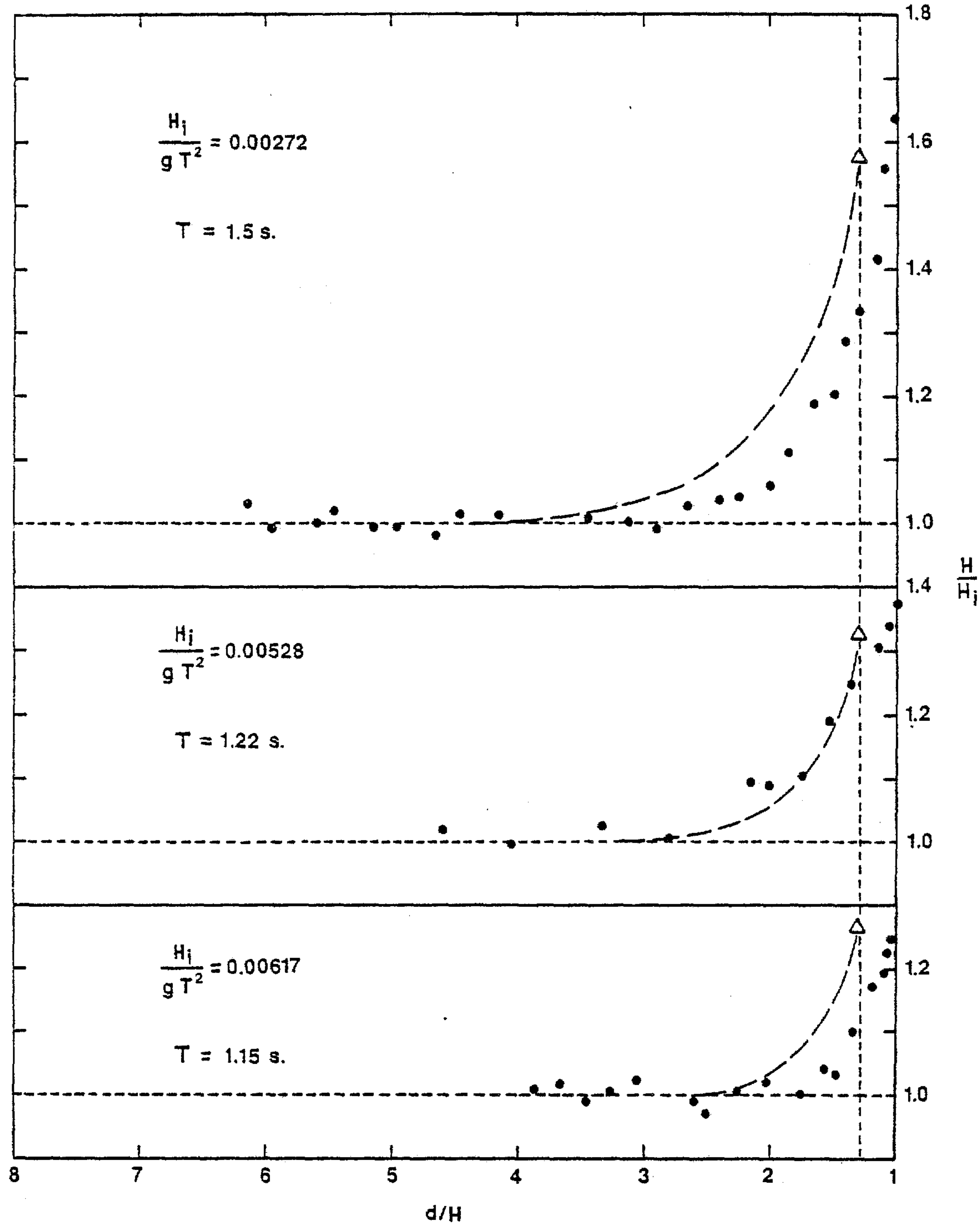


Figure 8a. (cont.)

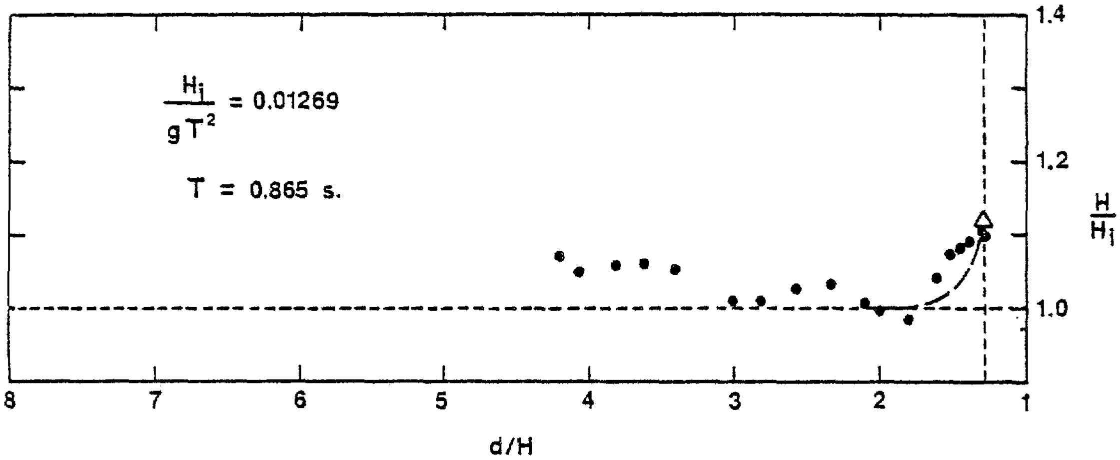


Figure 8a. (cont.)

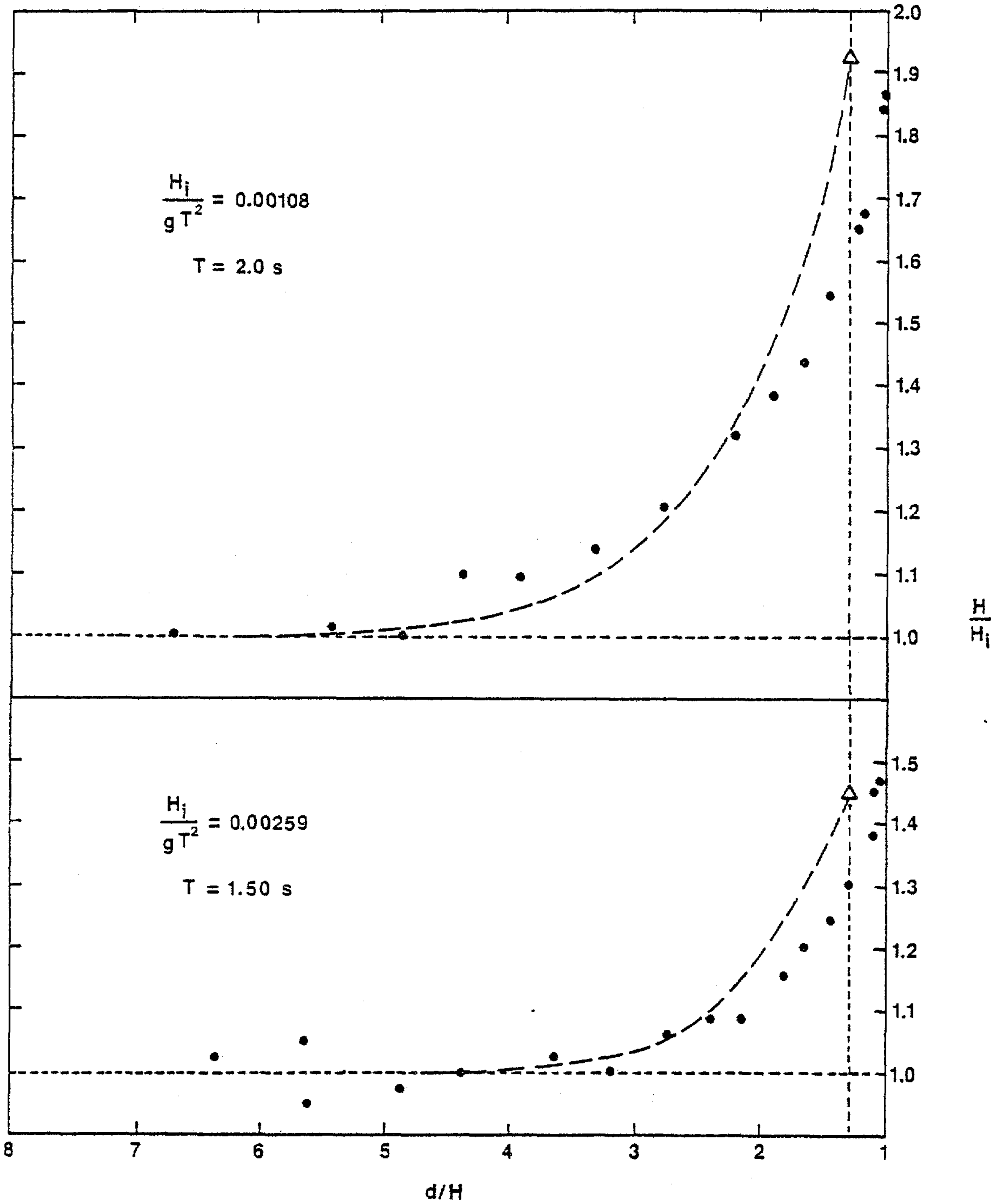


Figure 8b. Alpha wave peaking; data from Putnam (1945) for $\tan \alpha_b = 0.054$; shore-breaking occurs at Δ where d/H = 1.28.

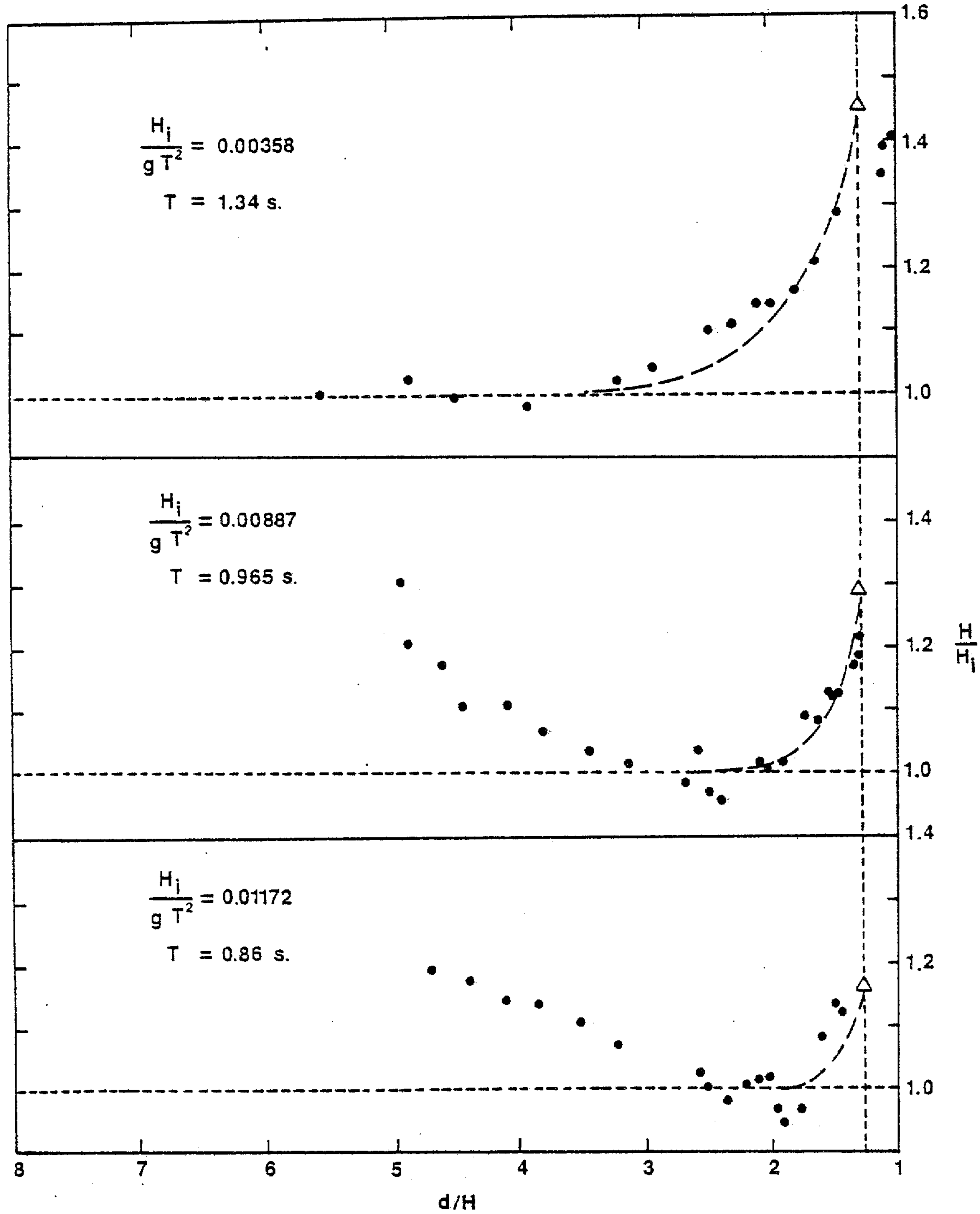


Figure 8b. (cont.)

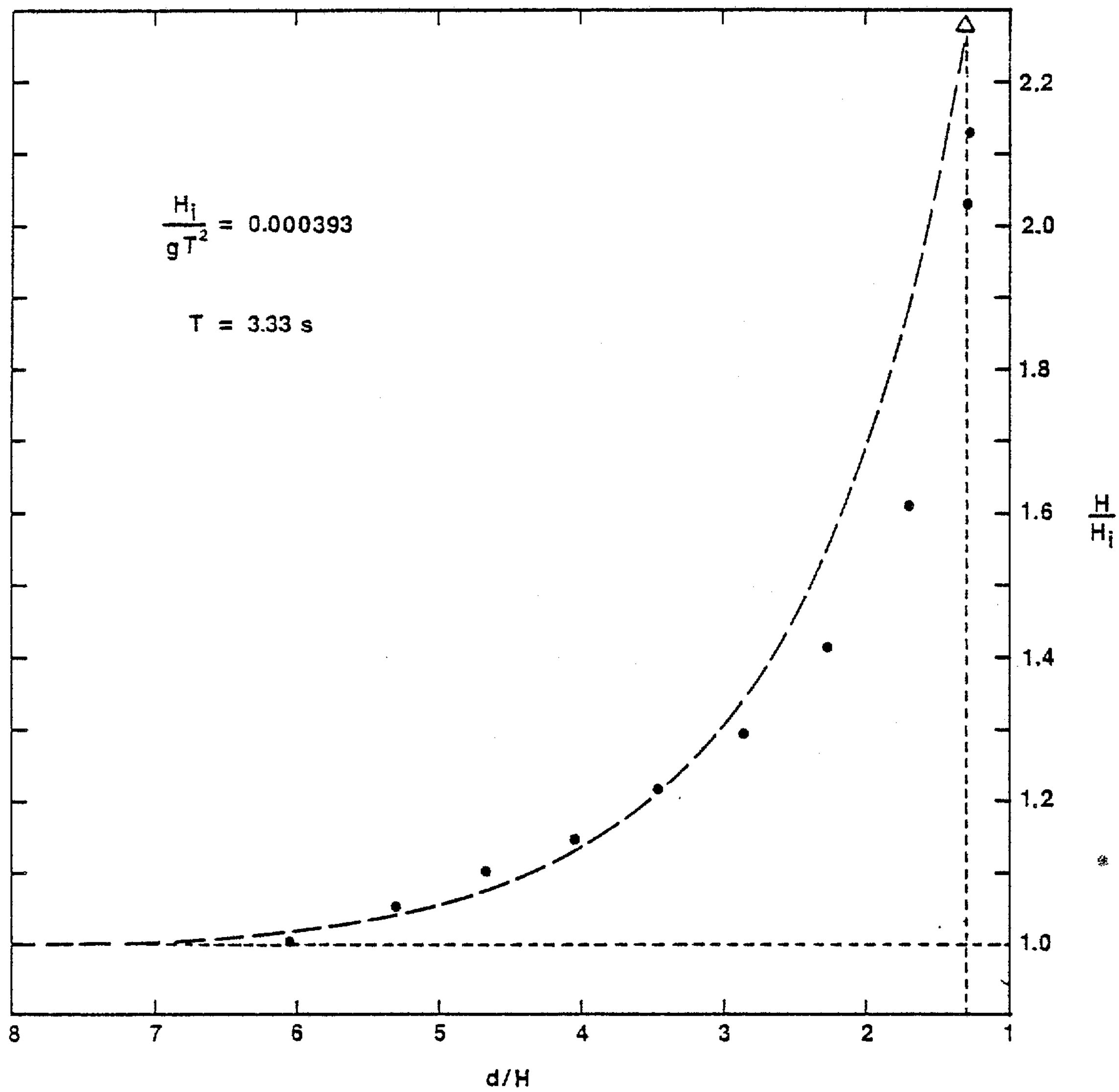


Figure 8c. Alpha wave peaking; data from Buhr Hansen and Svendsen (1979) for tan α_b = 0.0292; shore-breaking occurs at Δ where d/H = 1.28.

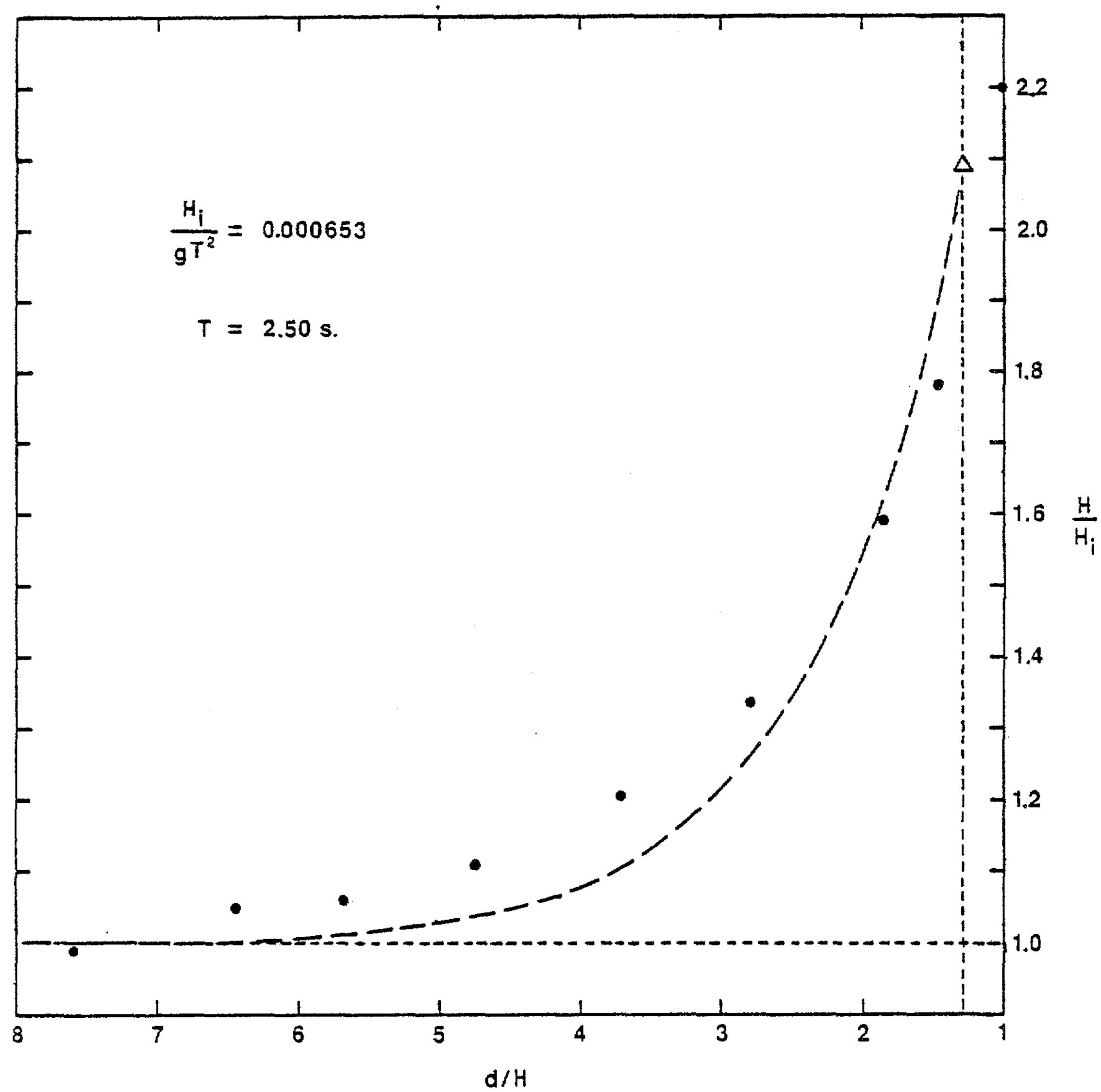


Figure 8c. (cont.)

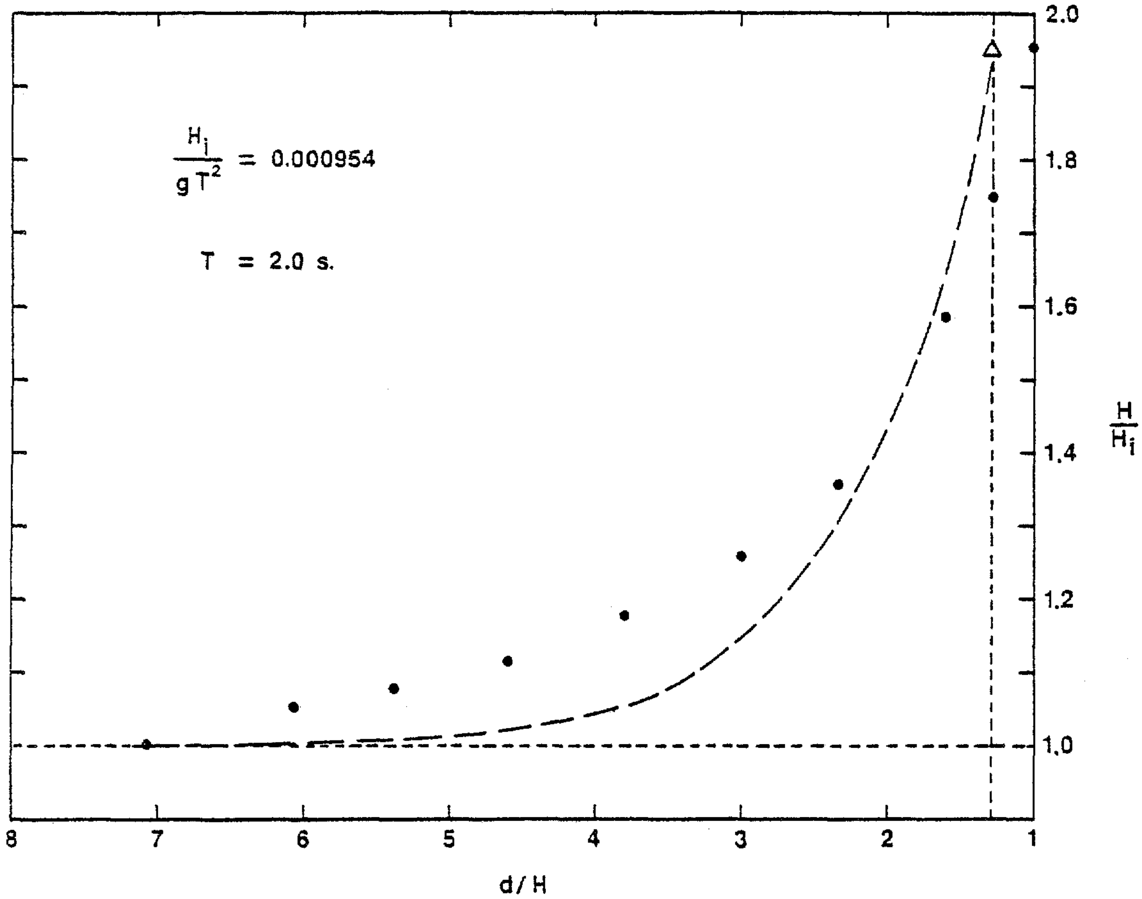


Figure 8c. (cont.)

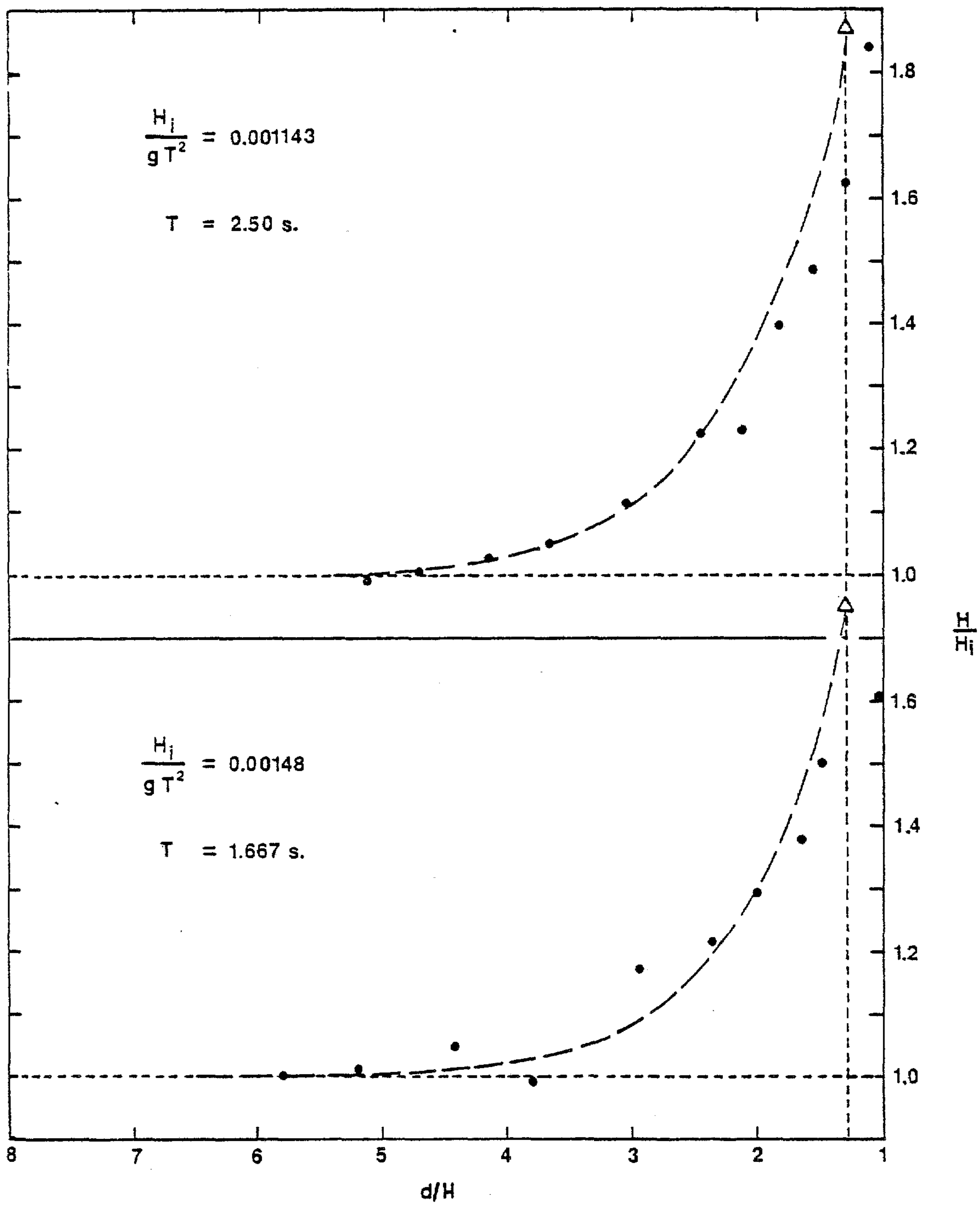
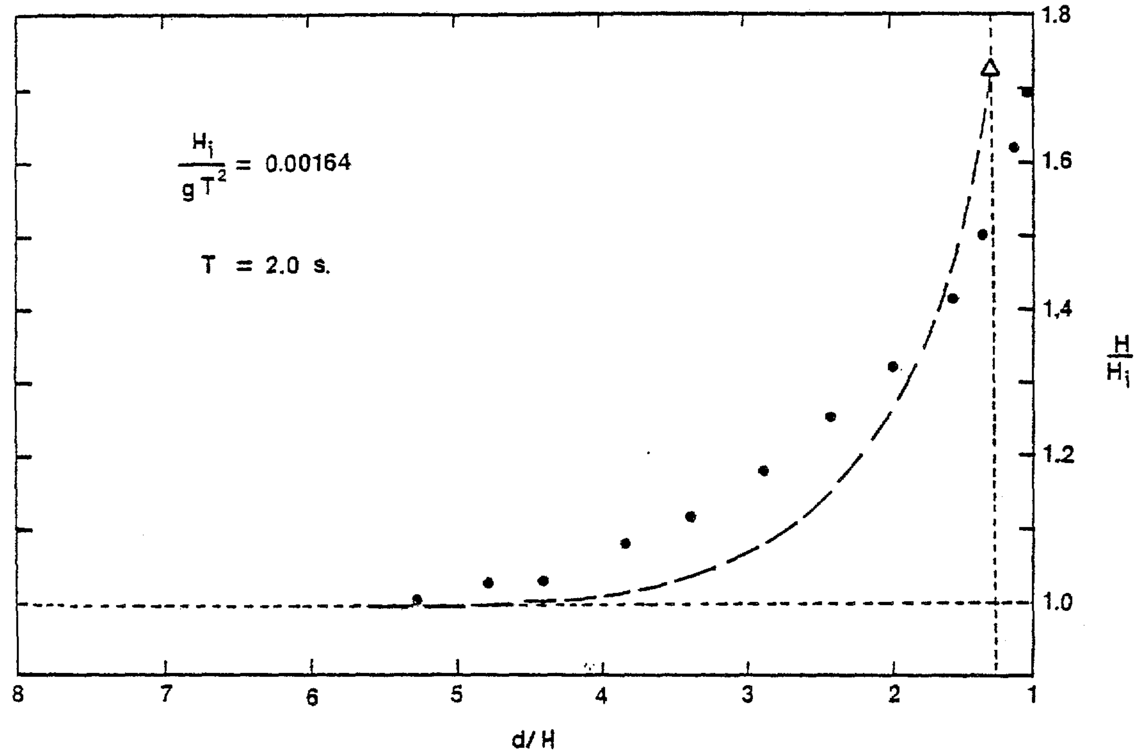


Figure 8c. (cont.)



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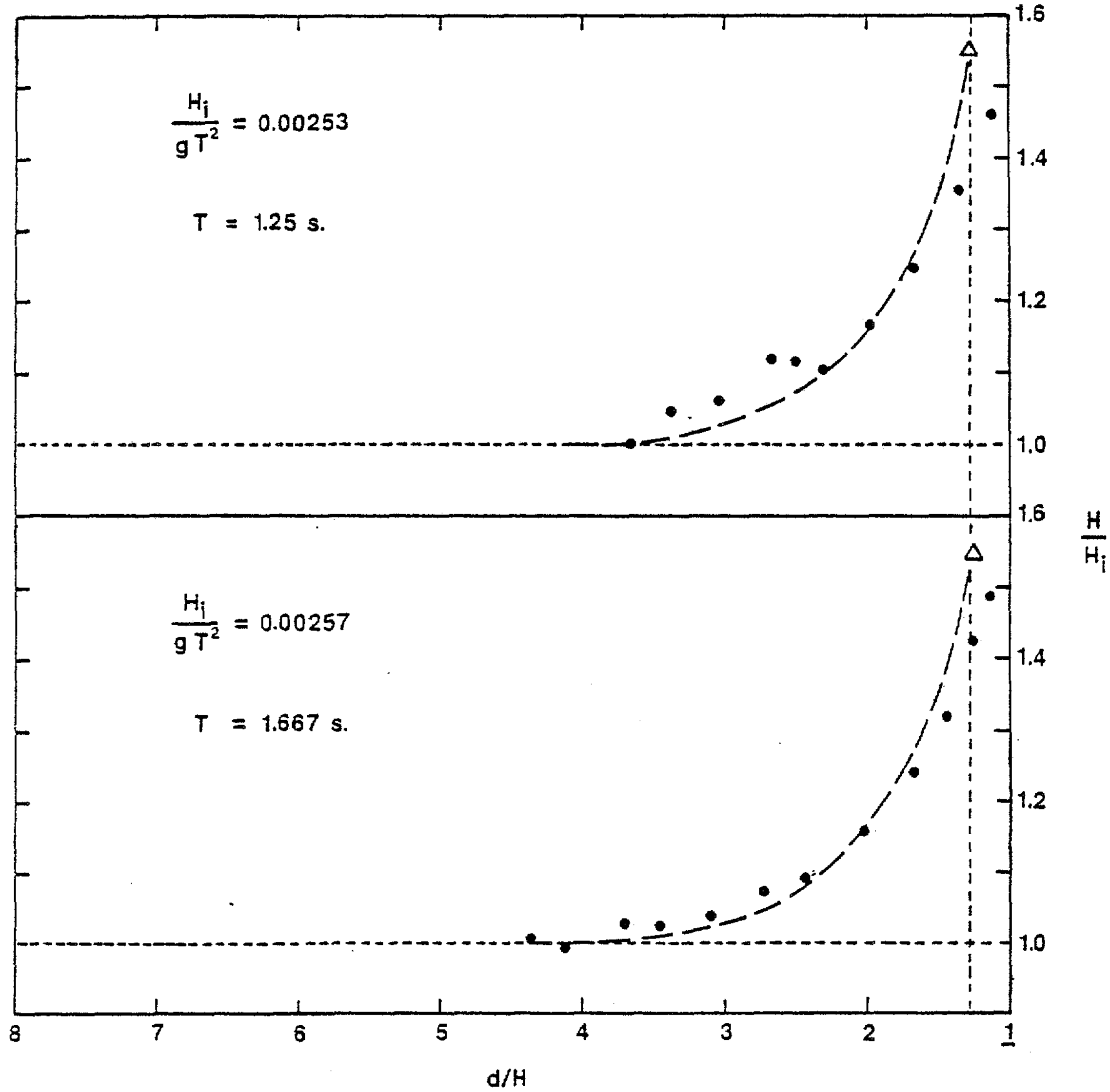

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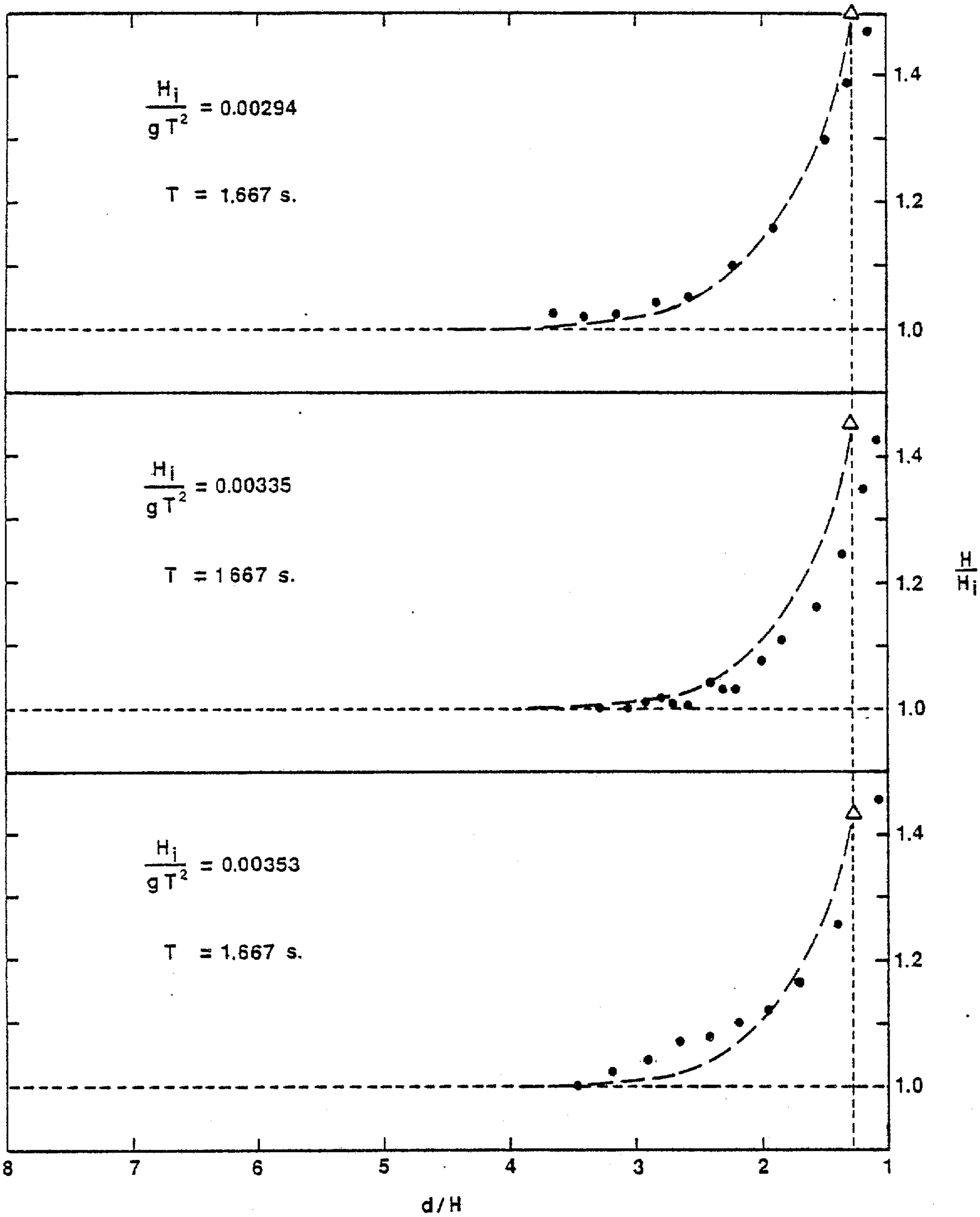

Figure 8c. (cont.)

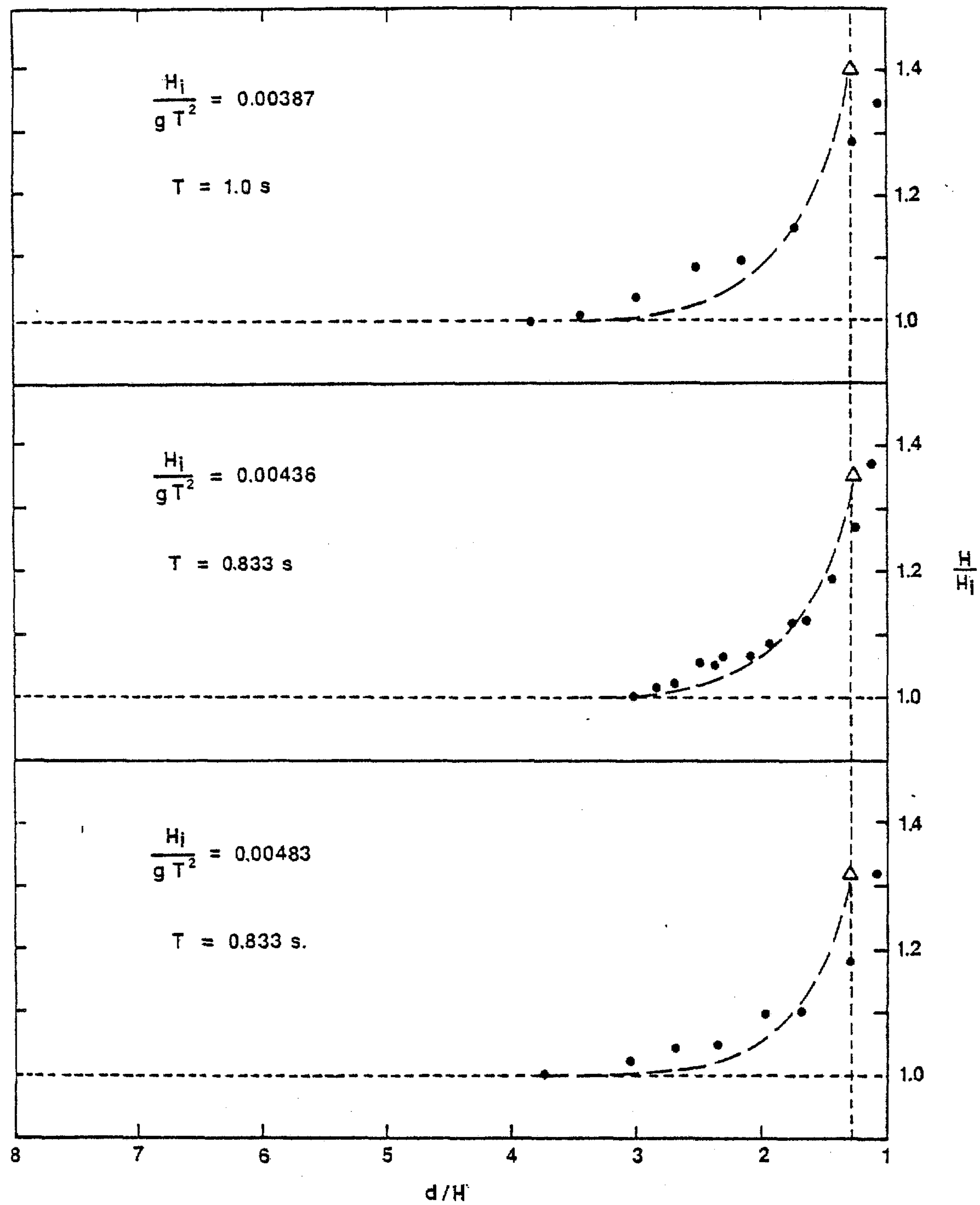

Figure 8c. (cont.)

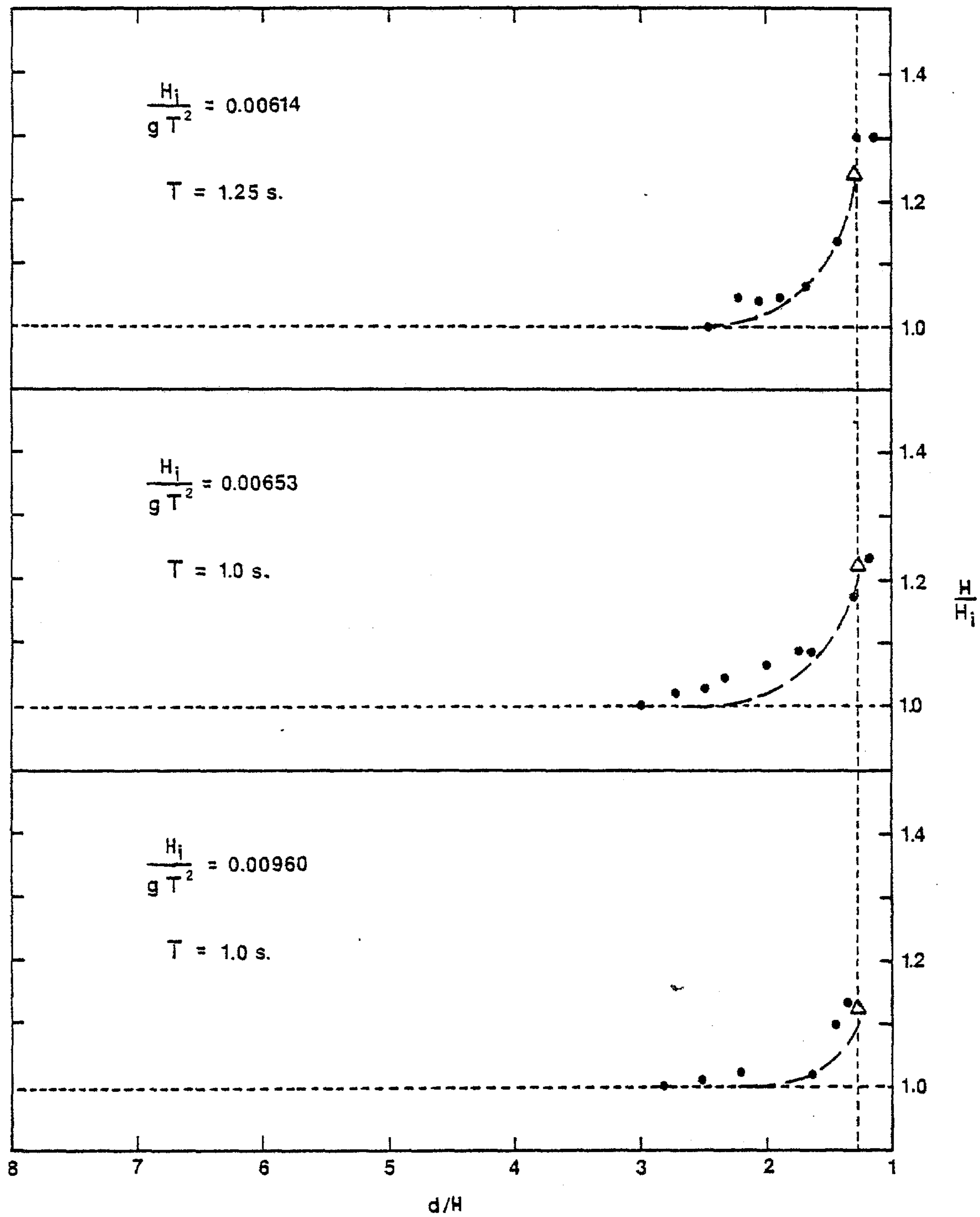

Figure 8c. (cont.)

in fact, it is seldom known. It has been the purpose of this paper, therefore, to provide a practicable solution to these problems, which represents wave height behavior during the shore-breaking process.

Two basic processes during shore-breaking have been identified as: 1. the total height of the wave tends to increase reaching a maximum at the shore-breaking point, and 2. the amount of the wave crest lying above the still water level tends to increase during the process. The latter process has been addressed in a companion paper (Balsillie, in manuscript). The former process defines the focus of attention in the present paper.

The point at which shore-breaking is initiated (i.e., incipient shore-breaking point) is given by equation (6), the wave height at the shore-breaking point by equation (2), and the wave height transformation by equation (7). The equations are dependent on the equivalent wave steepness parameter, $H/(g\ T^2)$, rather than d/L. Hence, dependence on the wave length is removed. The wave period is a readily available variable and, in addition, is conserved at least until shore-breaking occurs.

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